

UNIT-1Signal Analysis.Definition of Signal:

The signal is defined as a function of one (or) more independent variables which contains some information. The signal can be single dimensional (or) multidimensional.

→ When the function depends on single variable the signal is said to be one dimensional.

Ex: Voice [speech signal where amplitude wave is time]

→ When the function depends on 2 (or) more variables the signal is said to be multidimensional.

Ex: An image is an 2 dimensional signal ^{with} horizontal & vertical coordinates.

Definition of system:

The ~~set~~ system is set of elements (or) of functional blocks which are connected together to produce output in response of input.

 $x(t)$

System

 $y(t)$

$$y(t) = f(x(t))$$

$$y(t) = f(2t+3)$$

$$y(t) = 5$$

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Classification of signals:

The signals is broadly classified into '2' categories.

1. Analog signal.

2. Digital signal

Each of these analog & digital signals are further classified into as follows.

1. continuous & discrete time signals.

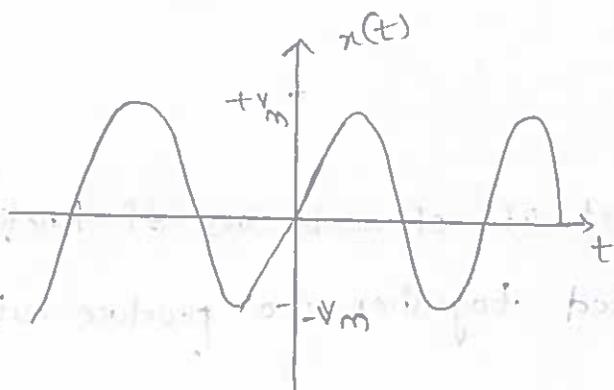
2. periodic & non-periodic signals.

3. even & odd signals.

4. Energy & power signals.

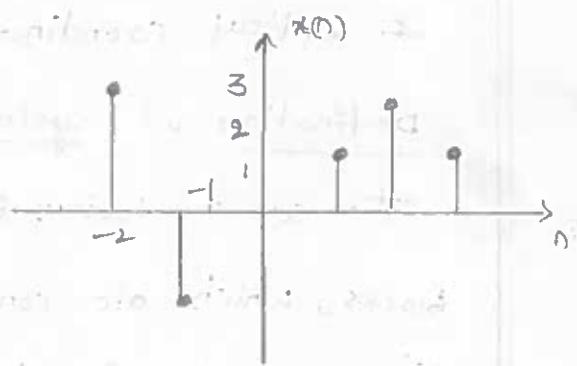
5. Random & deterministic signals.

Continuous & discrete time signals:



The signal $x(t)$ is said to be a continuous wave signal.

If $x(t)$ varies continuously with respect to time (t).



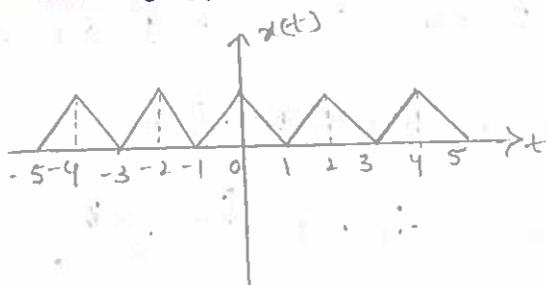
The signal $x(n)$ is said to be a discrete time signal.

If $x(n)$ exists only on discrete interval of time.

That is $x(n)$ exists particular interval of time.

Periodic signals

→ The continuous time signal $x(t)$ is said to be periodic signal with period (T) . If there is the non-zero value of T for which $x(t+T) = x(t)$ for all t .



For discrete Angular frequency

$$\omega = \frac{2\pi}{N}$$

$$N = \frac{2\pi}{\omega}$$

For continuous: $f_0 = 1/T_0$ Hz

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \text{ rad/sec}$$

For discrete time signal:

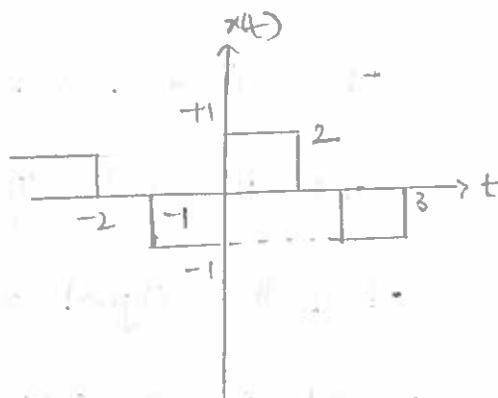
$$x(n+N) = x(n); \text{ for all } n$$

where N is a +ve integer & is called $x(n)$.

Note: The sum of two continuous periodic signals may not be periodic.

non-periodic signals.

→ The continuous time signal $x(t)$ is said to be non-periodic signal. If $x(t+T) \neq x(t)$ for all t .



* The sum of two periodic sequences is always periodic.

* The sum of 2 periodic signals is periodic only if the ratio of respective periods is a "rational number".

$$\therefore \frac{T_1}{T_2} = \text{rational number}$$

\therefore The ratio of 2 integers P_1, P_2

* The fundamental period is the LCM of P_1 & P_2 .

* If the ratio $\frac{T_1}{T_2}$ is an irrational number.

then the signal $x_1(t)$ & $x_2(t)$ do not have the common period & $x(t)$ cannot be periodic.

Determine whether the following signals are periodic or not if periodic, find its fundamental period.

1) $x(n) = \cos(3\pi n)$

Comparing the given eqⁿ $\cos 2\pi f n$

$$2\pi f n = 3\pi n$$

$$2f = 3$$

$$f = 3/2 = k/N \quad \text{the ratio of 2 integers}$$

Hence the given signal is periodic with $N=2$

2) $x(n) = \sin(3n)$

comparing the given Eqⁿ.

$2\pi f n = 3n$

$2\pi f = 3$

$f = 3/2\pi$

which is not a ratio of 2 integers

$x(n)$ is non-periodic.

3) $x(n) = \cos(0.01\pi n)$

comparing the given Eqⁿ.

$2\pi f n = 0.01\pi n$

$2f = 0.01$

$f = \frac{0.01}{2} = \frac{1}{200} = \frac{K}{N}; K=1, N=200$

hence the given signal is periodic with $N=200$.

4) $x(n) = e^{(j\pi/4)n}$

$e^{j\theta} = \cos\theta \pm j\sin\theta$

$= \cos(\pi/4)n \pm j\sin(\pi/4)n \rightarrow ①$

$= \cos 2\pi f n \pm j\sin 2\pi f n \rightarrow ②$

compare the above two Eqⁿs.

$2\pi f n = \pi/4 n$

$2f = 1/4$

$f = 1/8 = K/N$

hence the signal is periodic with $N=8$.

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$$5) \quad x(n) = \cos\left(\frac{2\pi n}{5}\right) + \cos\left(\frac{2\pi n}{7}\right)$$

comparing the given eqⁿ with standard eqⁿ.

$$\cos(2\pi f_1 n) + \cos(2\pi f_2 n)$$

$$2\pi f_1 n = \frac{2\pi n}{5} \Rightarrow f_1 = 1/5$$

$$= \frac{k_1}{N_1} \Rightarrow N_1 = 5$$

$$2\pi f_2 n = \frac{2\pi n}{7} \Rightarrow f_2 = 1/7$$

$$= \frac{k_2}{N_2} \Rightarrow N_2 = 7$$

$$\frac{N_1}{N_2} = 5/7 ; \quad \boxed{N = 35}$$

$$6) \quad x(n) = \cos\left(\frac{n\pi}{8}\right) + \cos\left(\frac{n\pi}{8}\right)$$

comparing the eqⁿ.

$$\cos 2\pi f_1 n + \cos n\pi f_2 n$$

$$2\pi f_1 n = n/8 \quad f_2 = 1/16$$

$$f_1 = \frac{1}{16\pi} \quad f_2 = \frac{1}{16}$$

→ Addition of periodic & non-periodic signal is non-periodic.

Even & odd signals:

A signal $x(t)$ or $x(n)$ is referred to as even signal.

if $x(-t) = x(t)$ (or) $x(-n) = x(n)$.

* An even signal have symmetric with respect to vertical axis.

* The signal for $t < 0$ is the mirror image of the signal $t > 0$.

* The function $x(t) = \cos \omega t$ is an even signal because $\cos(-\omega t) = \cos \omega t = x(t)$.

* A signal $x(t)$ (or) $x(n)$ is referred to as odd signal if $x(-t) = -x(t)$ (or) $x(-n) = -x(n)$.

* The signal for $t < 0$ is the mirror image of the signal for $t > 0$.

* The signal $x(t) = \sin \omega t$ is odd because

$$\begin{aligned} x(t) &= \sin \omega t \\ &= \sin(-\omega t) \\ &= -\sin \omega t \\ &= -x(t). \end{aligned}$$

1. Let us consider an arbitrary continuous time signals $x(t)$ here the objective is to find even & odd of this signal.

Let

$$x(t) = x_e(t) + x_o(t) \rightarrow 0$$

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where

$x_e(t) \rightarrow$ is the even part of $x(t)$

& $x_o(t) \rightarrow$ is the odd part of $x(t)$.

Since,

$x_e(t)$ is even we can write.

$$x_e(t) = x_e(-t) \rightarrow \textcircled{2}$$

Also,

$x_o(t)$ is odd we can write

$$x_o(-t) = -x_o(t) \rightarrow \textcircled{3}$$

put $t = -t$ in eqⁿ $\textcircled{1}$ we can get

$$x(-t) = x_e(-t) + x_o(-t) \rightarrow \textcircled{4}$$

Substituting $\textcircled{2}$ & $\textcircled{3}$ in $\textcircled{4}$ eqⁿ

$$x(-t) = x_e(t) - x_o(t) \rightarrow \textcircled{5}$$

Adding eqⁿ $\textcircled{1}$ & $\textcircled{5}$

$$x(t) + x(-t) = x_e(t) + x_o(t) + x_e(t) - x_o(t)$$

$$x(t) + x(-t) = 2x_e(t)$$

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)] \rightarrow \textcircled{6}$$

subtract eqⁿ $\textcircled{5}$ from $\textcircled{1}$

$$x(t) - x(-t) = x_e(t) + x_o(t) - x_e(t) + x_o(t)$$

$$x(t) - x(-t) = 2x_o(t)$$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)] \rightarrow \textcircled{7}$$

Similarly, $x_e(n) = \frac{1}{2} [x(n) + x(-n)]$

$$x_o(n) = \frac{1}{2} [x(n) - x(-n)]$$

Q, find the Even & odd components of the following, $x(t) = e^{jt}$

We know that
 $x(t) = x_e(t) + x_o(t)$.

Also $x_e(t) = \frac{1}{2} [x(t) + x(-t)]$
 $= \frac{1}{2} [e^{jt} + e^{-jt}]$
 $= \cos t$

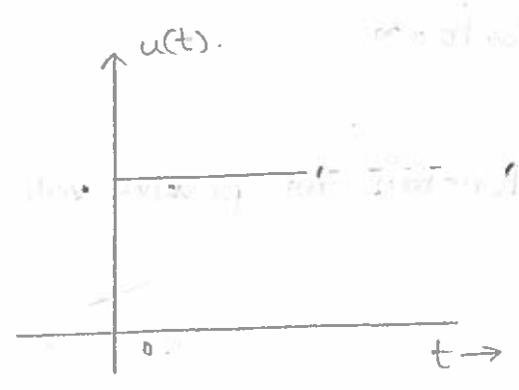
$x_e(t) = \cos t$.

$x_o(t) = \frac{1}{2} [x(t) - x(-t)] = \frac{1}{2} [e^{jt} - e^{-jt}]$
 $= j \sin t$
 $x_o(t) = j \sin t$.

Wkt $\frac{e^{j0} + e^{-j0}}{2} = \cos 0$

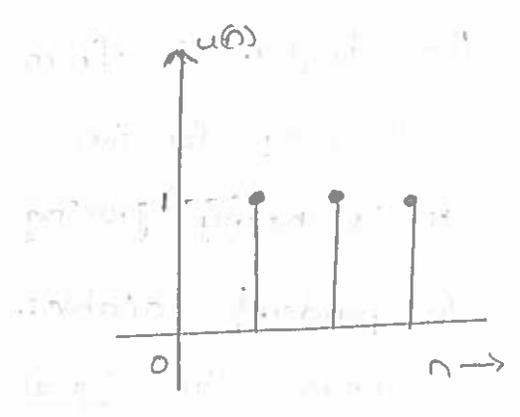
$\frac{e^{j0} - e^{-j0}}{2} = \sin 0$.

unit step function:-



$u(t) = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases}$

Continuous time signal



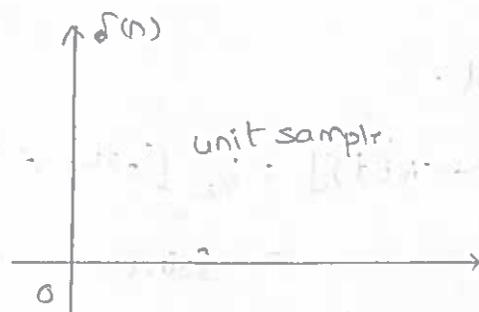
$u(n) = \begin{cases} 1; & n \geq 0 \\ 0; & n < 0 \end{cases}$

Discrete time signal.

Unit impulse (or) delta function:



$$\delta(t) = \begin{cases} 1; & t=0 \\ 0; & t \neq 0 \end{cases}$$



$$\delta(n) = \begin{cases} 1; & n=0 \\ 0; & n \neq 0 \end{cases}$$

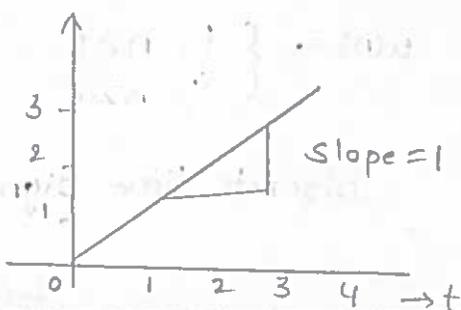
Note: $\delta(n)$ is not the sample version of $\delta(t)$

- * Area under $\delta(t) = 1$, amplitude of $\delta(n) = 1$
- * unit impulse or unit sample functions are used to determine impulse response of the system.
- * unit impulse or unit sample functions contain the all the frequencies from $-\infty$ to $+\infty$

Unit ramp function:

It is nearly growing function for positive values of independent variables.

continuous time signal:



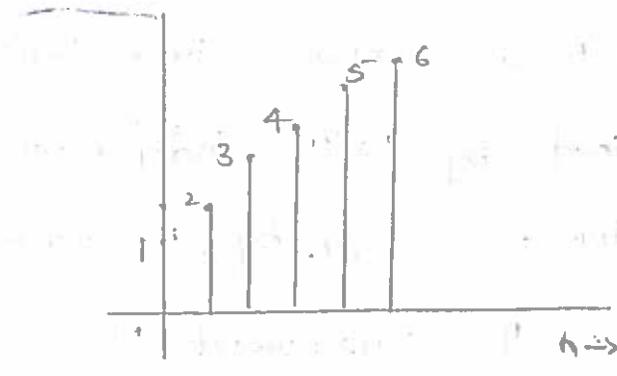
$$r(t) = \begin{cases} t; & t \geq 0 \\ 0; & t < 0 \end{cases}$$

$$r(t) = t u(t); \quad t \geq 0$$

$$\downarrow$$
$$u(t) = \begin{cases} 1; & t \geq 0 \end{cases}$$

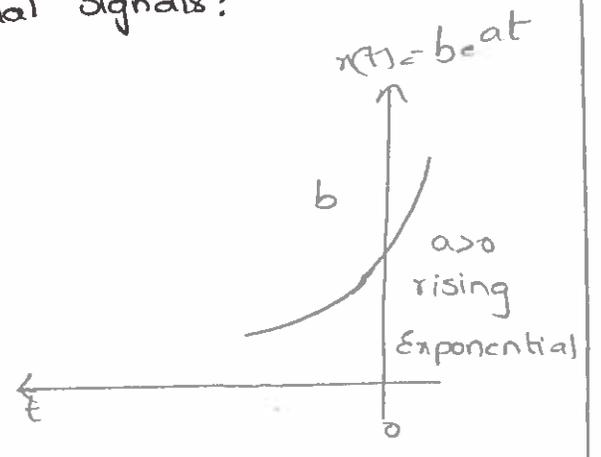
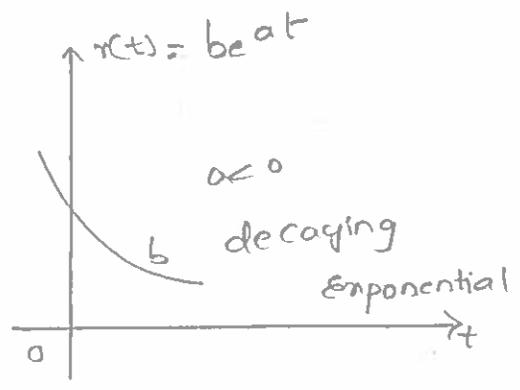
$$r(t) = t \times 1 = t, \quad t \geq 0.$$

Discrete time signal:



$$x(n) = \begin{cases} n; & n \geq 0 \\ 0; & n < 0 \end{cases}$$

Complex Exponential & Sinusoidal Signals:



Let $x(t) = be^{at}$; a & b are real

$$\begin{aligned} 2e^{-t} &= 2xe^{t3} \\ &= 2xe^{t2} \\ &= 2xe^1 \\ &= 2xe^0 \\ &= 2xe^{-1} \\ &= 2xe^{-2} \end{aligned}$$

$$\begin{aligned} 2xe^3 &= 40.17 \\ 2xe^2 &= 14.7 \\ 2xe^1 &= 5.4 \\ 2xe^0 &= 2 \\ 2xe^{-1} &= 0.67 \\ 2xe^{-2} &= 0.46 \end{aligned}$$

↓ decreasing.

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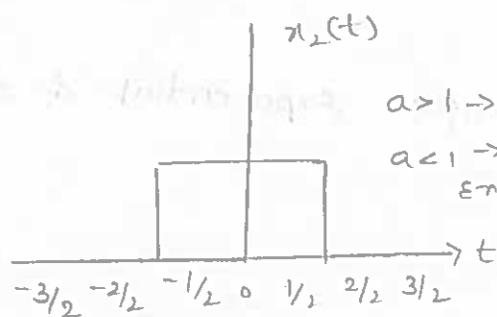
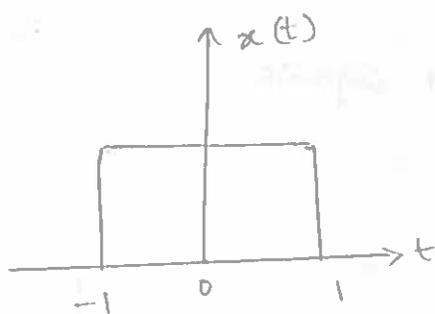
Unit-1, Pg-10/46

Operations performed on independent variables.

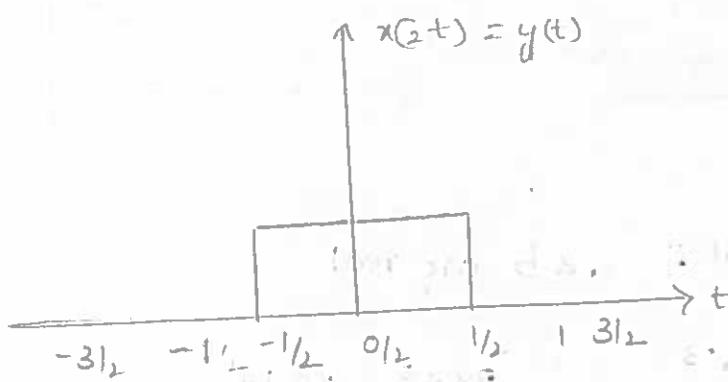
1, Time scaling: Let $x(t)$ be a continuous time signal, the signal $y(t)$ is obtained by scaling independent variable 't' by a factor a is given by $y(t) = x(at)$.

* When $a > 1$ then $y(t)$ is the compressed version of $x(t)$.

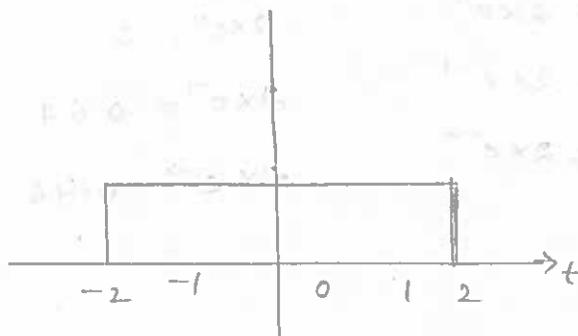
* When $a < 1$ then $y(t)$ is the expansion version of $x(t)$.



$a > 1 \rightarrow$ Compression
 $a < 1 \rightarrow$ Expansion



$$a = 1/2$$

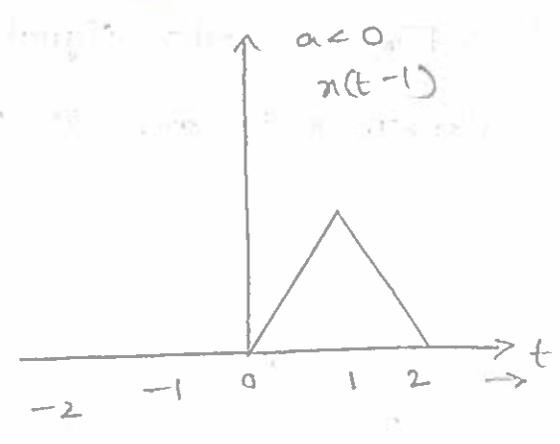
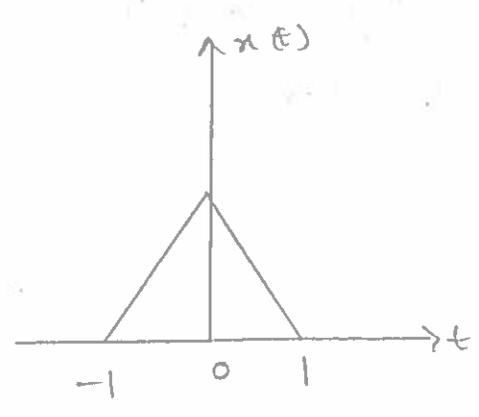


Time shifting:

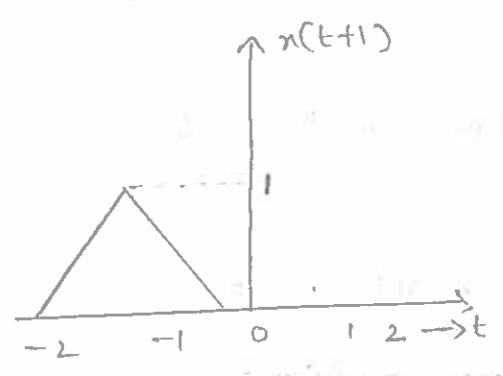
* let $x(t)$ be a continuous time signal, replacing $t \rightarrow t+a$ results in time shifted signal $y(t)$ defined by $y(t) = x(t+a)$

* If $a < 0$; $x(t)$ is shifted right by a secs.

if $a > 0$; $x(t)$ is shifted left by a secs.



$a > 0$; $y(t) = x(t+1)$



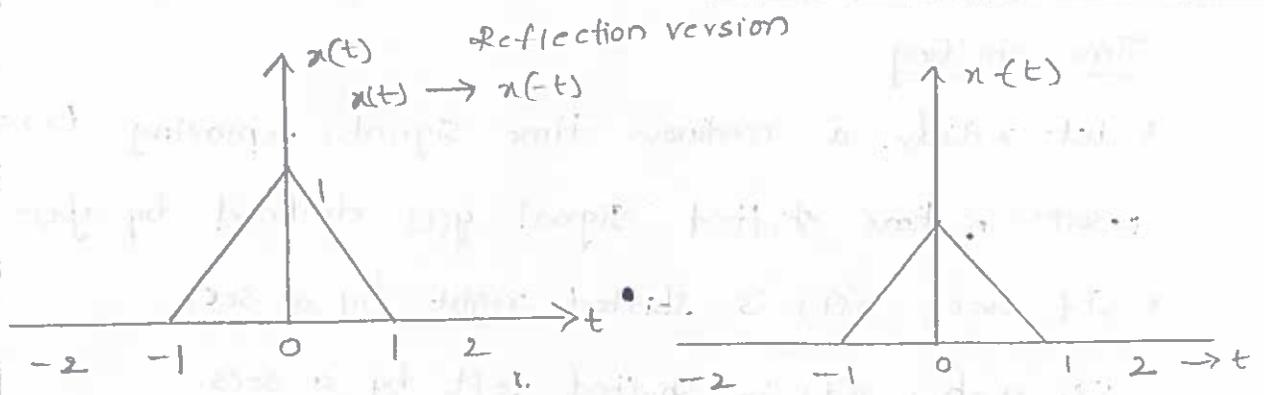
Reflection:-

consider a continuous time signal $x(t)$, the reflected version of $x(t)$ is obtained by replacing t with $-t$.

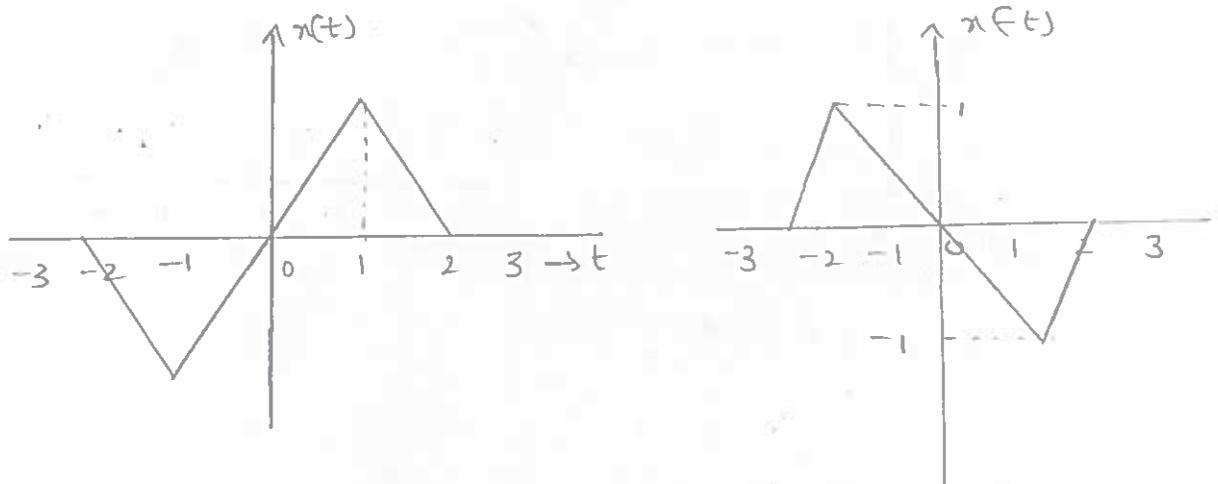
* The signal $x(t) = x(-t)$ represents of the reflection version of $x(t)$ about the vertical axis.

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Q) For an odd signal, the signal and its reflected version $x(-t)$ are $-ve$ of each other.



Precedence Rule for time shifting & scaling:

$$\text{let } y(t) = x(at+b).$$

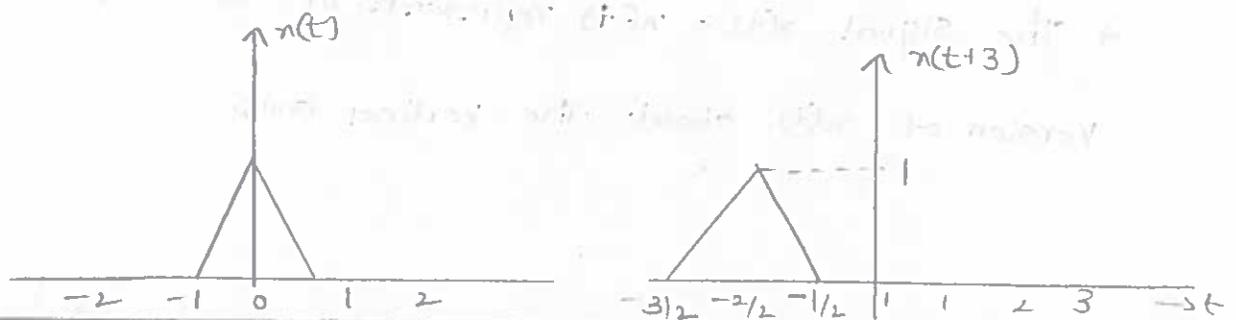
The above relation $y(t)$ & $x(t)$ must satisfies the following conditions.

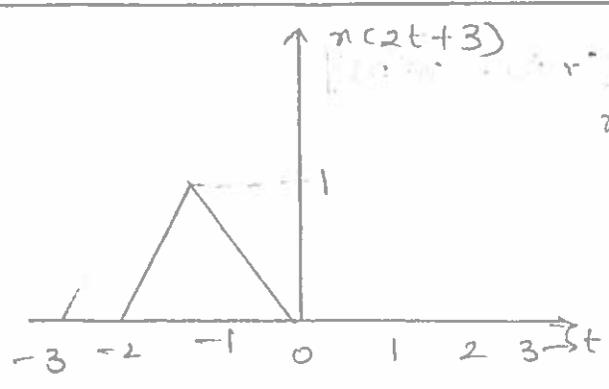
$$y(0) = x(b)$$

$$\& y(-b/a) = x(0).$$

$$x(at+b) \begin{cases} b=3 \\ a=2 \end{cases}$$

1) $y(t) = x(2t+3)$





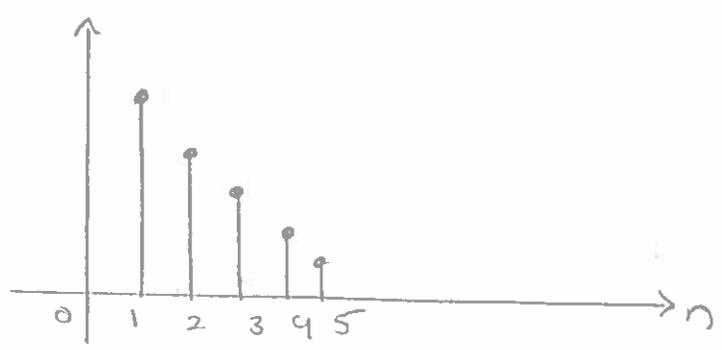
$x(t)$
 $x(2t) \Rightarrow a=2$

1) $x(n) = e^{(-n/4)} u(n)$; sketch even & odd parts.

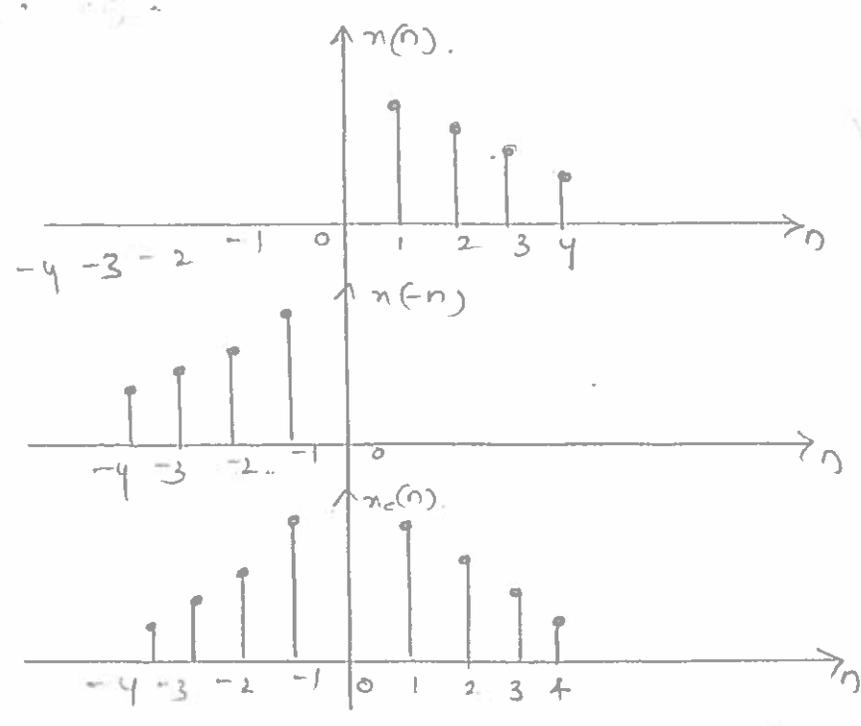
KKT, $u(n) = \begin{cases} 1; & n \geq 0 \\ 0; & n < 0 \end{cases}$

$x(0) = e^{(-0/4)} = 1$; $x(1) = e^{-1/4} = 0.778$.

$x(2) = e^{-2/4} = e^{-1/2} = 0.606$; $x(3) = e^{-3/4} = 0.47$.



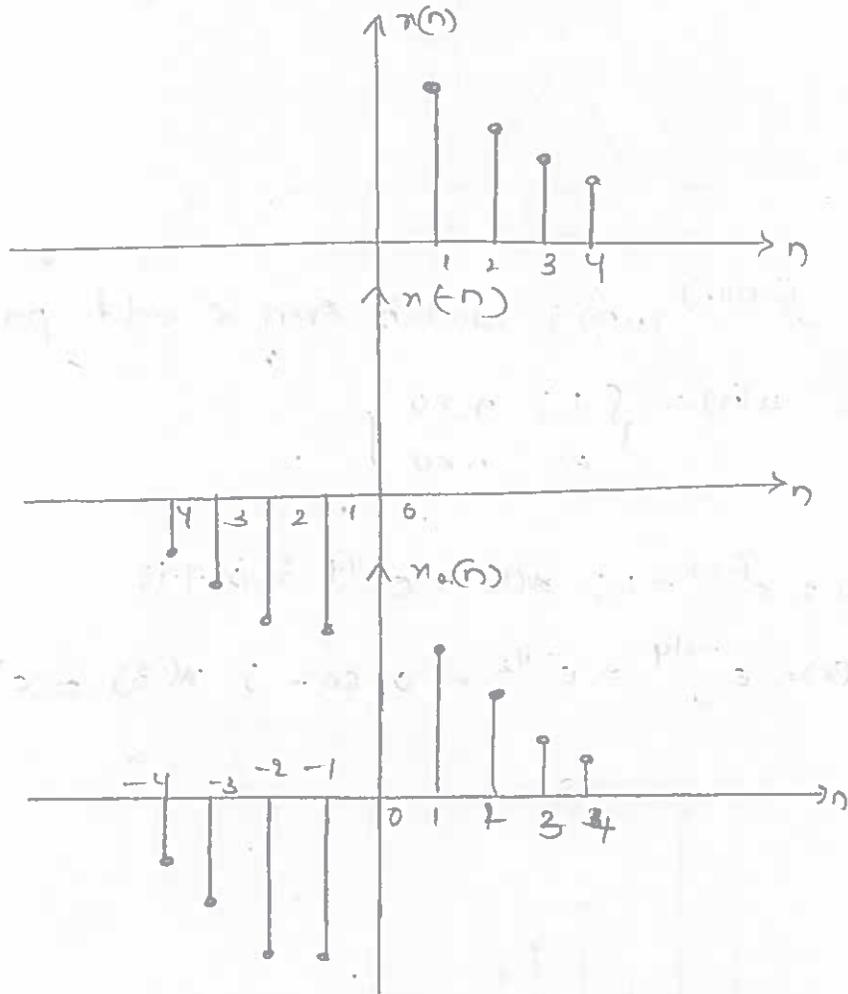
$x_e(n) = 1/2 [x(n) + x(-n)]$



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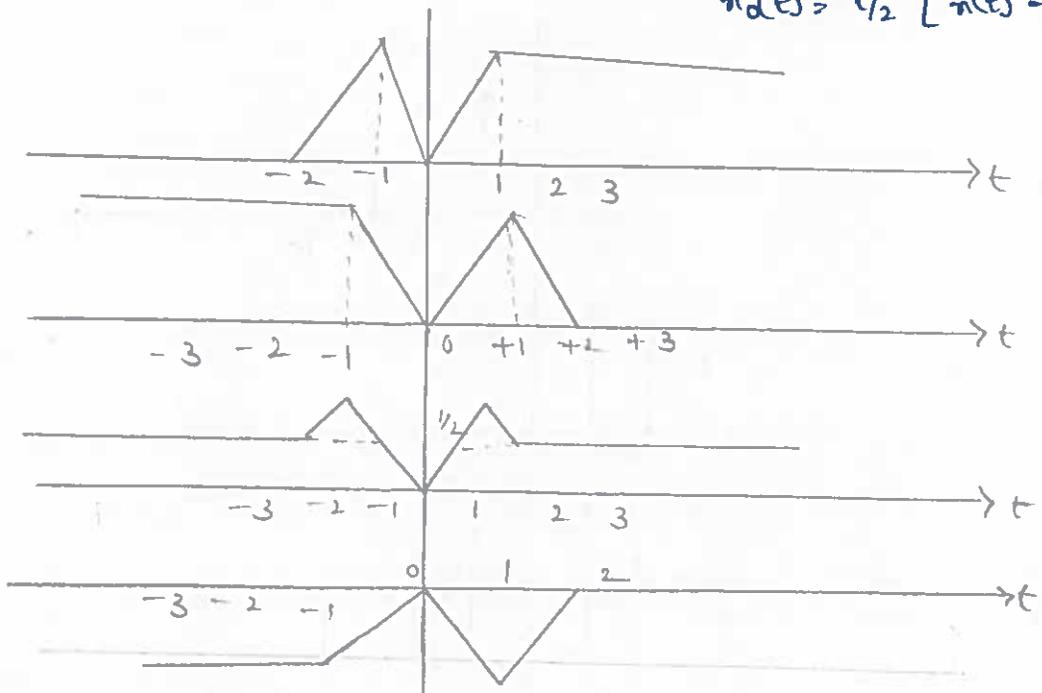
$$x_0(n) = \frac{1}{2} [x(n) - x(-n)]$$

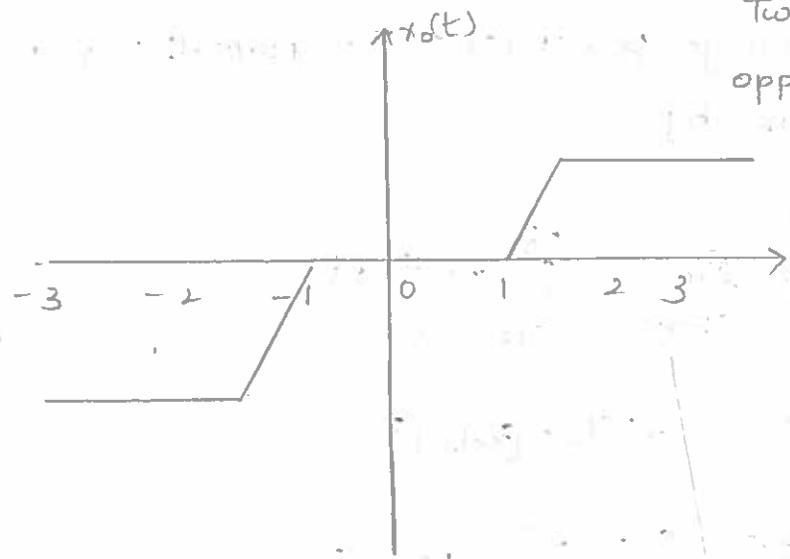


Find the even & odd parts:

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$





Energy & power signals:- consider an electrical circuit where voltage v of t is developed across a resistor are producing the current $i(t)$. The instantaneous power dissipated in the resistor is given by

$$p(t) = \frac{v^2(t)}{R} = i^2(t) R.$$

When the circuit consists of a unit resistance.

We get $p(t) = v^2(t) = i^2(t)$ (where $R=1$)

In general, if we consider a signal $x(t)$, where $x(t)$ may be a current (or voltage) signal.

Hence, the instantaneous power is given by

$$p(t) = x^2(t).$$

Then the total energy or normalized energy 'E' of the signal $x(t)$ is defined by

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T x^2(t) dt = \int_{-\infty}^{\infty} x^2(t) dt.$$

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The average power (or) the normalized average power is given by

CTS:

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt$$

Note: $x^2(t) = [x(t)]^2$

DTS: $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

Note:

1) Any signal whose total energy is finite are called Energy signal.

$$0 < E < \infty$$

2) They have zero power (deterministic, non periodic signals)

3) Any signal whose average power is finite are called power signals, they have infinite energy (random & period signal)

4) Both energy & power signals are mutually exclusive.

Determine whether the following signals are energy signals (or) power signals & calculate their energy (or) power.

1) $x(n) = (1/2)^n u(n)$.

$$u(n) = \begin{cases} 1 & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$$

$\therefore x(n)$ is non-periodic

\therefore It is an energy signal.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} |(1/2)^n u(n)|^2$$

$$E = \sum_{n=0}^{\infty} |(1/2)^n u(n)|^2 = \sum_{n=0}^{\infty} (1/4)^{n/2}$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} ; |r| < 1$$

$$\left(\frac{1}{2}\right)^0 \times 1 = 1$$

$$\left(\frac{1}{2}\right)^1 \times 1 = 1/2$$

$$\left(\frac{1}{2}\right)^2 \times 1 = 1/4$$

$$\left(\frac{1}{2}\right)^3 \times 1 = 1/8$$

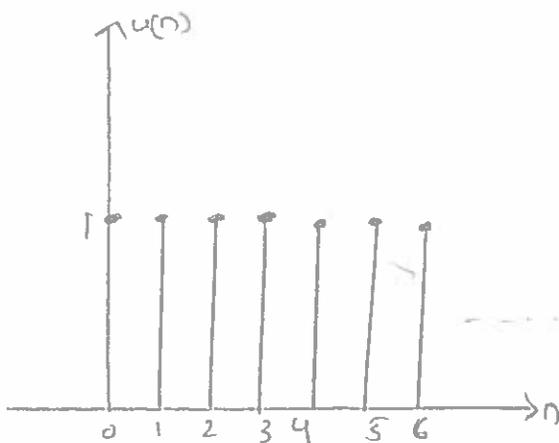
$$\left(\frac{1}{2}\right)^4 \times 1 = 1/16$$

$$E = \frac{1}{1-1/4} = \frac{4}{3} \text{ Joules}$$

2) $x(n) = u(n)$

We know that

$$u(n) = \begin{cases} 1 & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$$



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Here $u(n)$ repeats after every sample and is infinite duration hence it may be a power signal.

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |u(n)|^2$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} (1)^2 \quad \left[\because \sum_{n=0}^N (1)^2 = 1+1+\dots+N \right]$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} (N+1)$$

$$= \lim_{N \rightarrow \infty} \frac{N+1}{2N+1}$$

$$= \lim_{N \rightarrow \infty} \frac{N+1/N}{(2N+1)/N} = \lim_{N \rightarrow \infty} \frac{1+1/N}{2+1/N}$$

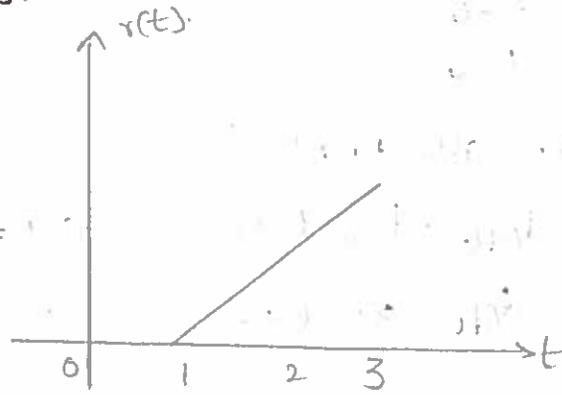
$$= \frac{1+1/\infty}{2+1/\infty} = \frac{1+0}{2+0} = 1/2$$

$$\boxed{P = 1/2 \text{ watts}}$$

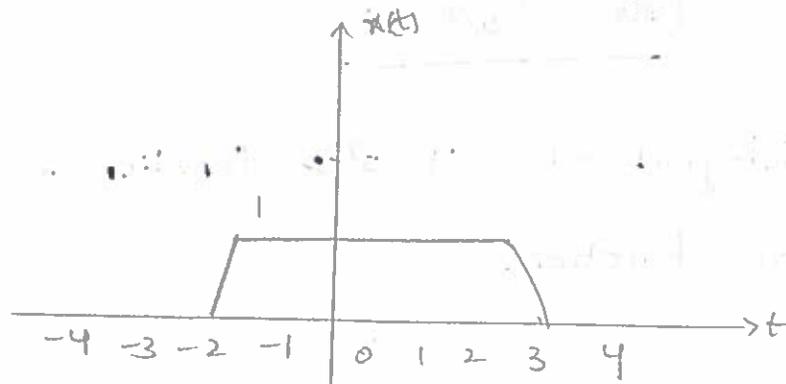
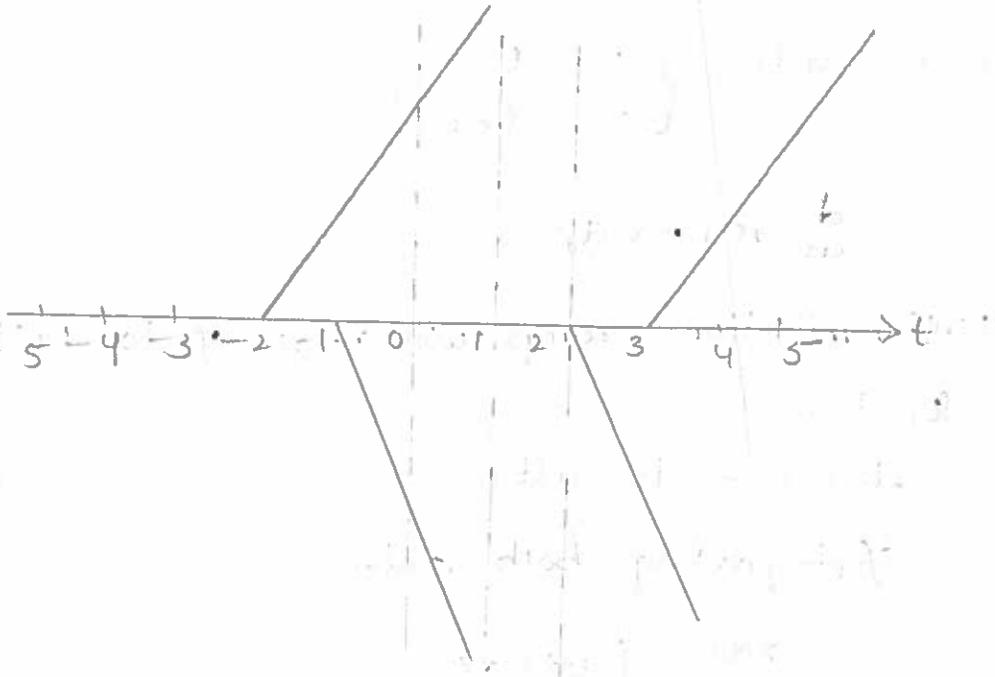
Note: unit step function is a power signal with

$$P = 1/2 \text{ W.}$$

Sketch the following signal & determine their even & odd components.



$$x(t) = \begin{cases} t; & t \geq 0 \\ 0; & t < 0 \end{cases}$$



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Relationship between unit step & unit ramp function:

$$r(t) = \begin{cases} t; & t \geq 0 \\ 0; & t < 0 \end{cases}$$

Differentiating $r(t)$ w.r.t 't'

$$\frac{d}{dt} r(t) = \begin{cases} \frac{d}{dt} \cdot t; & t \geq 0 \\ \frac{d}{dt} 0; & t < 0 \end{cases} = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases}$$

$$\frac{d}{dt} r(t) = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases}$$

Wkt, $u(t) = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases}$

$$\frac{d}{dt} r(t) = u(t)$$

Differentiation ramp will result into unit step function.

$$d \cdot r(t) = dt \cdot u(t)$$

Integrating both sides.

$$r(t) = \int u(t) dt$$

$$\boxed{r(t) = \int u(t) dt}$$

Integral of unit step function result in to ramp function.

relation between unit step & impulse function

$$\left(\frac{d}{dt} u(t) = \delta(t) \right)$$

$$\int \delta(t) dt = u(t)$$

Find & sketch the derivatives of

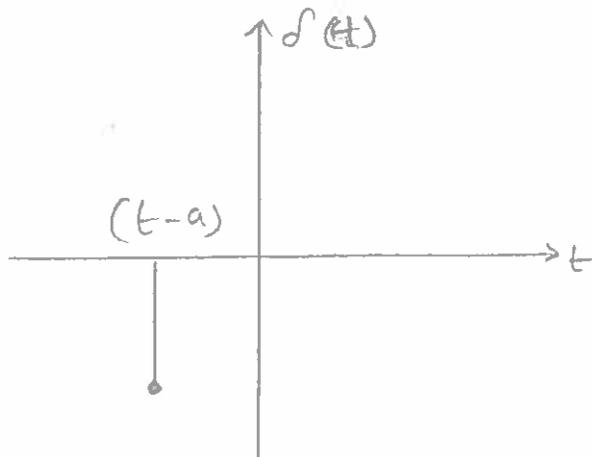
$$x(t) = u(t) - u(t-a); a > 0$$

$\begin{cases} a > 0; \text{left} \\ a < 0; \text{right} \end{cases}$

$$\frac{d}{dt} x(t) = \frac{d}{dt} [u(t) - u(t-a)]; a > 0$$

$$= \frac{d}{dt} u(t) - \frac{d}{dt} u(t-a); a > 0$$

$$\frac{d}{dt} x(t) = \delta(t) - \delta(t-a); a > 0$$



Find & sketch the derivative of $x(t) = [t u(t) - u(t-a)]$

$$\frac{d}{dt} x(t) = \frac{d}{dt} [t u(t) - u(t-a)]$$

$$\text{let } y(t) = u(t) - u(t-a)$$

$$\frac{d}{dt} y(t) = \frac{d}{dt} u(t) - \frac{d}{dt} u(t-a)$$

$$\frac{d}{dt} y(t) = \delta(t) - \delta(t-a)$$

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$$\frac{d}{dt} x(t) = \frac{d}{dt} t y(t).$$

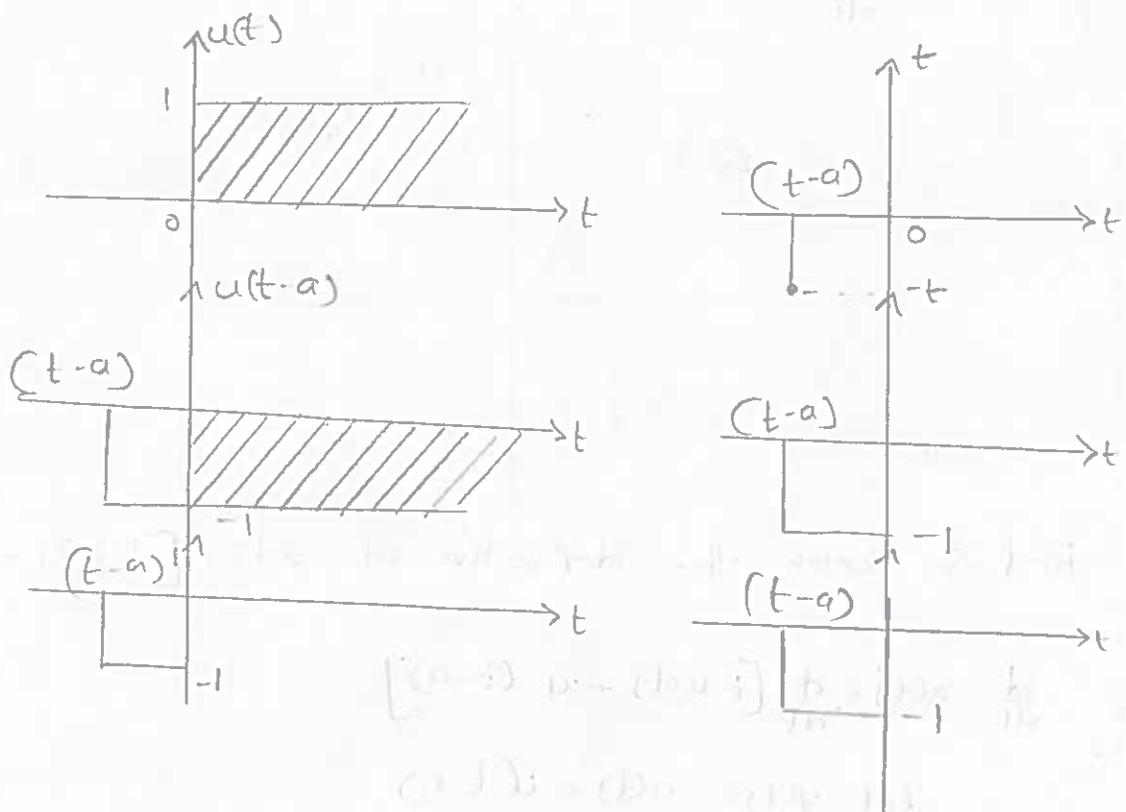
$$= t \frac{dy(t)}{dt} + y(t) \frac{dt}{dt}$$

$$= t \frac{dy(t)}{dt} + y(t)$$

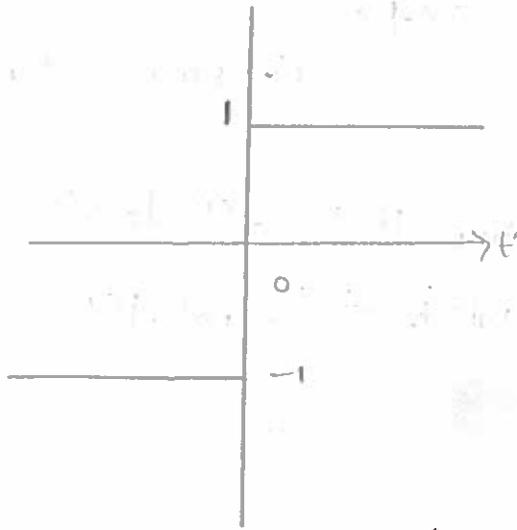
$$= t \cdot [\delta(t) - \delta(t-a)] + u(t) - u(t-a)$$

$$\frac{d}{dt} x(t) = t \delta(t) - t \delta(t-a) + u(t) - u(t-a);$$

$$u(t) = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases} \quad / \quad u(t-a) = \begin{cases} 1; & (t-a) \geq 0 \\ 0; & (t-a) < 0 \end{cases}$$



Signum function



$$\text{sgn}(t) = \begin{cases} 1; & t \geq 0 \\ -1; & t < 0 \\ 0; & t = 0 \end{cases}$$

$$= 2u(t) - 1$$

- Analog between vectors & signals.

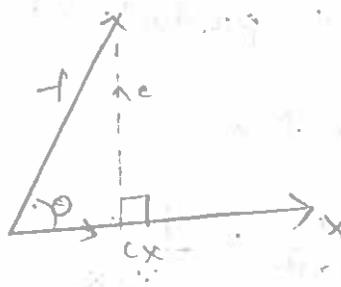
signals \rightarrow orthogonal functions.

orthogonality concept in vectors:

- All signals are basically vectors. Where vector is represented in terms of its co-ordinates, consider a vector \vec{r} & another vector \vec{x} then projection of vector along ^{other} vector is shown below.

The dot product of \vec{r} & \vec{x} is given as

$$\vec{r} \cdot \vec{x} = |\vec{r}| |\vec{x}| \cos \theta$$



$\vec{r} \cos \theta$ is the component of vector \vec{r} along \vec{x}

(or)

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projection of \vec{f} on \vec{x}

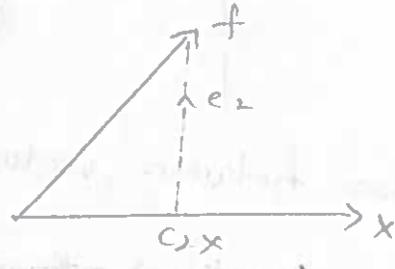
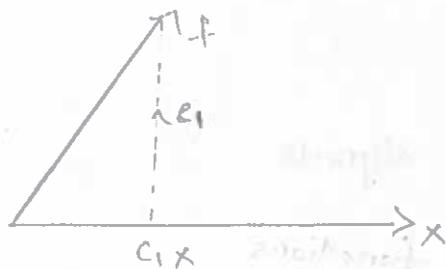
using vector addition $\vec{f} = c\vec{x} + \vec{e}$

(\vec{e} - error vector)

Note:

\vec{e} is the minimum only when it is \perp to \vec{x}

Below figures show in which \vec{e} is not \perp



$$\vec{f} = c_1\vec{x} + \vec{e}_1 = c_2\vec{x} + \vec{e}_2 \quad (\text{here } e_1 \text{ \& } c_2 \text{ are greater than } e)$$

The component of \vec{f} along \vec{x} is called $c\vec{x}$

which is given by $f \cos \theta$

$$c|\vec{x}| = |\vec{f}| \cos \theta$$

Multiplying both sides by \vec{x}

$$c|\vec{x}|^2 = |\vec{f}| |\vec{x}| \cos \theta$$

dot product of vectors \vec{f} & \vec{x}

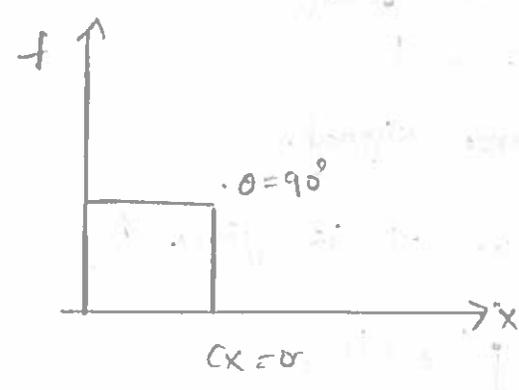
$$c|\vec{x}|^2 = \vec{f} \cdot \vec{x}$$

$$c = \frac{\vec{f} \cdot \vec{x}}{|\vec{x}|^2} = \frac{\vec{f} \cdot \vec{x}}{\vec{x} \cdot \vec{x}}$$

($\because \vec{x} \cdot \vec{x}$ & $\vec{f} \cdot \vec{x}$ are vector products & $\vec{x} \cdot \vec{x}$

cannot get cancelled)

When 'f' is \perp to 'x', f will not have component along 'x' because $\theta = 90^\circ$ as shown in fig:



$$f \cdot x = |f| |x| \cos \theta$$

since $\theta = 90^\circ$

$$f \cdot x = |f| |x| \cos 90^\circ$$

$$f \cdot x = 0 \quad (\cos 90^\circ = 0)$$

The vectors 'f' & 'x' are said to be 'orthogonal', if their dot product is zero.
(or)

vectors are orthogonal, if they are mutually \perp .

orthogonality in signals:

consider a signal $f(t)$ to be represented in terms of $x(t)$ over an interval t_1 & t_2 .

$$f(t) \longrightarrow x(t), \quad t_1 \text{ to } t_2$$

$$f(t) = c_1 x(t) + e(t)$$

(or)

$$e(t) = f(t) - c_1 x(t); \quad t_1 \leq t \leq t_2 \longrightarrow \text{or}$$

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Energy of $e(t)$ is given by

$$E = \int_{t_1}^{t_2} e^2(t) dt \quad \text{--- (1)}$$

Energy of error signal.

Mean square value of $e(t)$ is given by

$$\overline{e^2(t)} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} e^2(t) dt$$

$$\overline{e^2(t)} = \frac{E_e}{t_2 - t_1}$$

\therefore From eqⁿ (1) we can write eqⁿ (2)

$$E_e = \int_{t_1}^{t_2} [f(t) - c x(t)]^2 dt$$

Here the value of c , should be selected such

that E_e will be minimum.

\rightarrow This can be obtained by differentiating E_e with c and equating it to zero.

for E_e to be minimum

$$\frac{dE_e}{dc} = 0$$

$$\frac{d}{dc} \left[\int_{t_1}^{t_2} (f(t) - c x(t))^2 dt \right] = 0$$

$$\frac{d}{dc} \left[\int_{t_1}^{t_2} f(t) dt - 2 \int_{t_1}^{t_2} f(t) c x(t) + \int_{t_1}^{t_2} c^2 x^2(t) dt \right] = 0$$

$$= -2 \int_{t_1}^{t_2} f(t) x(t) dt + 2c \int_{t_1}^{t_2} x^2(t) dt = 0$$

$$2c \int_{t_1}^{t_2} x^2(t) dt = \int_{t_1}^{t_2} f(t) x(t) dt$$

$$c = \frac{\int_{t_1}^{t_2} f(t) x(t) dt}{\int_{t_1}^{t_2} x^2(t) dt}$$

The same expression can be obtained for minimum value of \bar{e}^2

In the above eqn denominator represents energy of $x(t)$ which cannot be zero.

Hence, numerator must be zero to make c zero
 $c = 0$

If $c = 0$, there will be no component of $f(t)$ along $x(t)$, $f(t)$ & $x(t)$ is said to be orthogonal over an interval t_1, t_2 $[t_1, t_2]$

i.e., $\int_{t_1}^{t_2} f(t) x(t) dt = 0$

Similarly, if $f(t)$ & $x(t)$ are complex signals then they are orthogonal over an interval (t_1, t_2) if any one of the component is conjugate.

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$$\int_{t_1}^{t_2} f(t) x^*(t) dt = 0 \quad \text{con} \quad \int_{t_1}^{t_2} f^*(t) x(t) dt = 0$$

\downarrow \downarrow
 complex conjugate of $x(t)$. Complex conjugate of $f(t)$.

1) Show that the followings are orthogonal over the interval (0,1).

$$f(t) = 1; \quad x(t) = \sqrt{3} (1-2t)$$

Q.10] B.K.T $\int_{t_1}^{t_2} f(t) x(t) dt = 0$

$$= \int_0^1 1 \cdot \sqrt{3} (1-2t) dt = 0$$

$$= \int_0^1 \sqrt{3} dt - \int_0^1 2\sqrt{3} t dt = 0$$

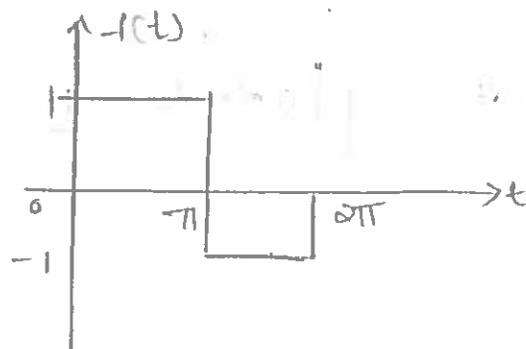
$$= \sqrt{3} (t)_0^1 - 2\sqrt{3} \left[\frac{t^2}{2} \right]_0^1 = 0$$

$$= \sqrt{3} (1-0) - \sqrt{3} (1-0) = 0$$

$$= \sqrt{3} - \sqrt{3} = 0$$

The two signals are orthogonal.

Below fig shows square wave represent the signal by
 sint plot an error in this representation.



Let square wave be $f(t)$ & sin wave be $x(t) = \sin t$
 then $f(t) = c x(t)$

$$f(t) = c \sin t$$

$$c = \frac{\int_a^b f(x) x(t) dt}{\int_a^b x^2(t) dt}$$

$$= \int_a^b f(t) x(t) dt = \int_0^{2\pi} f(t) \sin t dt$$

$$= \int_0^{\pi} 1 \cdot \sin t dt + \int_{\pi}^{2\pi} (-1) \sin t dt$$

$$= \int_0^{\pi} \sin t dt + \int_{\pi}^{2\pi} -\sin t dt$$

$$= [-\cos t]_0^{\pi} + [\cos t]_{\pi}^{2\pi}$$

$$= -[\cos \pi - \cos 0] + [\cos 2\pi - \cos \pi]$$

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$$= -[(-1) + (1)]$$

$$= 2 + 2 = 4$$

$$\int_a^b x^2 \cos x \, dx = \int_0^{2\pi} \sin^2 x \, dx \quad \left[\int \sin^2 x \, dx = \frac{1 - \cos 2x}{2} \right]$$

$$= \int_0^{2\pi} \frac{1 - \cos 2x}{2} \, dx$$

$$= \int_0^{2\pi} \frac{1}{2} \, dx - \int_0^{2\pi} \frac{\cos 2x}{2} \, dx$$

$$= \frac{1}{2} (x)_0^{2\pi} - \frac{1}{2} \left[\frac{\sin 2x}{2} \right]_0^{2\pi}$$

$$= \frac{1}{2} (2\pi - 0) - \frac{1}{4} (\sin 4\pi - \sin 0)$$

$$= \pi - \frac{1}{4} [0 - 0]$$

$$= \pi$$

$$C = \frac{\int_a^b f(x) \, dx}{\int_a^b x^2 \, dx}$$

$$\left(C = \frac{4}{\pi} \right)$$

$$\therefore f(x) = C \sin x$$

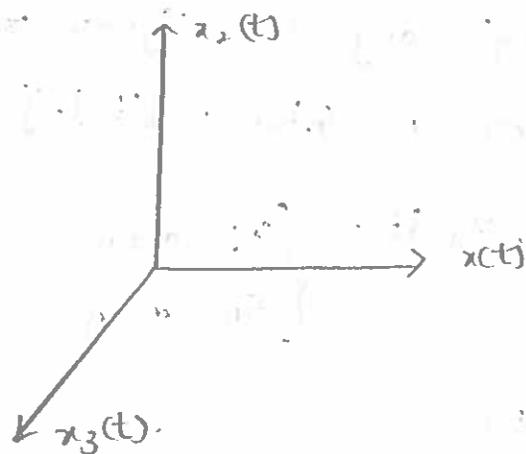
$$f(x) = \frac{4}{\pi} \sin x$$

$$e = f(x) - Cx(x)$$

$$= \frac{4}{\pi} \sin x - \frac{4}{\pi} \sin x$$

$$\left(e = 0 \right)$$

orthogonal signal space.



Let $x_1(t)$, $x_2(t)$ & $x_3(t)$ be orthogonal to each other
 i.e. three signals are mutually \perp to each other.
 which forms three dimensional signal space, which is
 also called as orthogonal signal space.

this is used to represent any signal lying in that
 space.

Note: If there are n such mutually orthogonal signal
 i.e. $x_1(t)$, $x_2(t)$, $x_3(t)$... $x_n(t)$ then they form n
 dimensional orthogonal signal space.

Signal approximation using orthogonal functions

Consider a set of signals which are mutually orthogonal
 over an interval $[t_1, t_2]$.

$f(t)$ can be represented as $f(t) \approx c_1 x_1(t) + c_2 x_2(t) +$
 $c_3 x_3(t) + \dots + c_N x_N(t)$

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$$\therefore f(t) \approx \sum_{n=1}^N c_n x_n(t) + e(t) \rightarrow \text{①}$$

In the above eqⁿ any two signals $x_m(t)$ & $x_n(t)$ are orthogonal over an interval $[t_1, t_2]$

$$\text{i.e., } \int_{t_1}^{t_2} x_m(t) x_n(t) dt = \begin{cases} 0; & m \neq n \\ E_n; & m = n \end{cases}$$

Because if $m=n$,

$$\int_{t_1}^{t_2} x_n(t) x_n(t) dt = \int_{t_1}^{t_2} x_n^2(t) dt = E_n$$

Energy of the signal.

Error $e(t)$ in the approximation of eqⁿ is given as

$$e(t) = f(t) - \sum_{n=1}^N c_n x_n(t)$$

Hence Error Energy is given by $E_e(t) = \int_{t_1}^{t_2} e^2(t) dt$

$$E_e = \int_{t_1}^{t_2} \left[f(t) - \sum_{n=1}^N c_n x_n(t) \right]^2 dt$$

Hence E_e will be minimized w.r.t c_i if

$$\frac{\partial E_e}{\partial c_i} = 0$$

$$\frac{\partial}{\partial c_i} \left[\int_{t_1}^{t_2} \left(f(t) - \sum_{n=1}^N c_n x_n(t) \right)^2 dt \right] = 0$$

$$\frac{\partial}{\partial c_i} \left[\int_{t_1}^{t_2} f^2(t) dt - 2 \int_{t_1}^{t_2} f(t) \sum_{n=1}^N c_n x_n(t) dt + \int_{t_1}^{t_2} \sum_{n=1}^N (c_n x_n(t))^2 dt \right]$$

for $i=1, 2, 3, \dots, N$

The eqn is executed

The first integration term is independent of c_i

So, its derivative is zero.

$$\therefore \frac{\partial}{\partial c_i} \left[-2 \int_a^{t_2} c_i f(t) \varphi_i(t) dt + \int_a^{t_2} c_i^2 \varphi_i^2(t) dt \right] = 0$$

$$= -2 \int_a^{t_2} f(t) \varphi_i(t) dt + 2 c_i \int_a^{t_2} \varphi_i^2(t) dt = 0$$

$$c_i = \frac{\int_a^{t_2} f(t) \varphi_i(t) dt}{\int_a^{t_2} \varphi_i^2(t) dt} \quad ; \quad i=1, 2, 3, \dots, N$$

$$c_i = \frac{1}{\int_a^{t_2} \varphi_i^2(t) dt} \int_a^{t_2} f(t) \varphi_i(t) dt$$

Mean Square Error:

$$E_e = \int_a^{t_2} e^2(t) dt = \int_a^{t_2} \left[f(t) - \sum_{n=1}^N c_n \varphi_n(t) \right]^2 dt$$

$$E_e = \int_a^{t_2} f^2(t) dt - 2 \int_a^{t_2} \sum_{n=1}^N c_n f(t) \varphi_n(t) dt + \int_a^{t_2} \sum_{n=1}^N c_n^2 \varphi_n^2(t) dt$$

Integration & Summation order if we interchange, we get,

$$E_e = \int_a^{t_2} f^2(t) dt - 2 \sum_{n=1}^N c_n \int_a^{t_2} f(t) \varphi_n(t) dt + \sum_{n=1}^N c_n^2 \int_a^{t_2} \varphi_n^2(t) dt$$

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$$E_e = \int_{t_1}^{t_2} f^2(t) dt - 2 \sum_{n=1}^N c_n c_n \epsilon_n + \sum_{n=1}^N c_n^2 \epsilon_n$$

$$E_e = \int_{t_1}^{t_2} f^2(t) dt - \sum_{n=1}^N c_n^2 \epsilon_n$$

The mean square error & error energy are related as

$$\overline{e^2(t)} = \frac{E_e}{t_2 - t_1} = \frac{1}{t_2 - t_1} \left[\int_{t_1}^{t_2} f^2(t) dt - \sum_{n=1}^N c_n^2 \epsilon_n \right]$$

$\therefore \sum_{n=1}^N c_n^2 \epsilon_n$ is always positive, so if

$$E_e \rightarrow 0 \text{ as } N \rightarrow \infty$$

closed (or) complete set of orthogonal function

Mean square error approaches to zero as number

of terms $c_n^2 \epsilon_n$ are made infinite under the condition

with $\overline{e^2(t)} = 0$ as $N \rightarrow \infty$.

$$0 = \frac{1}{(t_2 - t_1)} \left[\int_{t_1}^{t_2} f^2(t) dt - \sum_{n=1}^N c_n^2 \epsilon_n \right]$$

$$\therefore \int_{t_1}^{t_2} f^2(t) dt = \sum_{n=1}^N c_n^2 \epsilon_n$$

$$f(t) = \sum_{n=1}^N c_n \phi_n(t)$$

$$f(t) = \sum_{n=1}^{\infty} c_n \phi_n(t) \rightarrow \text{Generalized Fourier series.}$$

It is said to be complete (or) closed set if there

exist no function $p(t)$ for which $\int_{t_1}^{t_2} p(t) \phi_n(t) dt = 0$

$n = 1, 2, 3, \dots$

If $p(t)$ exists & above integral is zero, then $p(t)$ must be member of set $\{x_n(t)\}$ for complete set function. $f(t)$ is expressed as $c_1 x_1(t) + c_2 x_2(t) + c_3 x_3(t) + \dots$

$$c_i = \frac{\int_{t_1}^{t_2} f(t) x_i(t) dt}{\int_{t_1}^{t_2} x_i^2(t) dt} \quad ; \quad i = 1, 2, 3, \dots, N$$

$$c_i = \frac{1}{E_i} \int_{t_1}^{t_2} f(t) x_i(t) dt$$

orthogonality for complex functions:

Let us consider a set of signals $x_1(t), x_2(t), x_3(t), \dots, x_N(t)$. are complex, then those signals are mutually orthogonal, if

$$\int_{t_1}^{t_2} x_m(t) x_n^*(t) dt = \int_{t_1}^{t_2} x_m^*(t) x_n(t) dt$$

$$\begin{cases} 0; & m \neq n \\ E_n; & m = n \end{cases}$$

but $f(t) = \sum_{n=1}^N c_n x_n(t)$.

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$$\text{where } c_n = \frac{1}{E_n} \int_{t_1}^{t_2} f(t) x_n^*(t) dt$$

$$\& E_n = \int_{t_1}^{t_2} x_n(t) x_n^*(t) dt.$$

If $x(t)$ & $y(t)$ are orthogonal then show that the energy of the signal $x(t) + y(t)$ is identical to the energy of the signal $x(t)$ + the energy of the signal $y(t)$.

Q8] Let the energy of the signal $x(t)$ be E_x & $y(t) \rightarrow E_y$

$$\text{ie, } E_x = \int_{-\infty}^{\infty} x^2(t) dt ; E_y = \int_{-\infty}^{\infty} y^2(t) dt$$

Energy of sum of signals $x(t)$ & $y(t)$ will be

$$\int_{-\infty}^{\infty} [x(t) + y(t)]^2 dt$$

$$= \int_{-\infty}^{\infty} [x^2(t) + 2x(t)y(t) + y^2(t)] dt$$

$$= \int_{-\infty}^{\infty} x^2(t) dt + \int_{-\infty}^{\infty} y^2(t) dt + \int_{-\infty}^{\infty} 2x(t)y(t) dt$$

Since $x(t)$ & $y(t)$ are orthogonal third integration term in the above eqn will be zero.

$$\int_{-\infty}^{\infty} [x(t) + y(t)]^2 dt = \int_{-\infty}^{\infty} x^2(t) dt + \int_{-\infty}^{\infty} y^2(t) dt$$

$$\int_{-\infty}^{\infty} [x(t) + y(t)]^2 dt = E_x + E_y.$$

∴ Sum of energies of orthogonal signals = energy of the total sum of signals.

Show that the signals set $\{1, \cos \omega_0 t, \cos 2\omega_0 t, \dots, \cos n\omega_0 t, \dots, \sin \omega_0 t, \sin 2\omega_0 t, \dots, \sin n\omega_0 t\}$ are orthogonal over an interval $T_0 = \frac{2\pi}{\omega_0}$

Sol ∴ To check orthogonality of cosine waves

Consider the orthogonality of $\cos n\omega_0 t$ $\cos m\omega_0 t$

$$\int_t^{t+T_0} \cos n\omega_0 t \cos m\omega_0 t dt$$

$$\cos n \cos y = \frac{1}{2} (\cos(n-y) + \cos(n+y))$$

$$\int_t^{t+T_0} \cos n\omega_0 t \cos m\omega_0 t dt = \frac{1}{2} [\cos(n-m)\omega_0 t dt] + \frac{1}{2} [\cos(n+m)\omega_0 t dt]$$

for $n=m$

$\cos(n-m)\omega_0 t = 1$, but for $n \neq m$.

The integration of $(n-m)$ full cycle of cosine wave is taken over one period.

Hence, integration is zero

Similarly, integration of $n+m$ full cycle.

$$\therefore \int_t^{t+T_0} \cos n\omega_0 t \cos m\omega_0 t dt = \frac{1}{2} \int_t^{t+T_0} 1 \cdot dt$$

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$$= \frac{1}{2} (t) \Big|_t^{t+T_0}$$

$$= \frac{1}{2} (t+T_0 - t) = \frac{T_0}{2}$$

$$\therefore \int_t^{t+T_0} \cos n\omega t \cos m\omega t dt = \begin{cases} 0; & n \neq m \\ T_0/2; & n = m \end{cases}$$

Two cosine waves of given set are orthogonal over one period.

ii) To check orthogonality of sine waves

$$\int_t^{t+T_0} \sin n\omega t \sin m\omega t dt$$

$$\int \sin n \sin m dt = \frac{1}{2} [\cos(n-m) - \cos(n+m)]$$

$$\int_t^{t+T_0} \sin n\omega t \sin m\omega t dt = \frac{1}{2} [\cos(n-m)\omega t - \cos(n+m)\omega t]$$

$$\text{if } n=m, \cos(n-m)\omega t = 1$$

(Integration of cosine wave over one period is zero)

$$\text{then } \int \cos(n+m)\omega t dt = 0$$

$$\int_t^{t+T_0} \sin n\omega t \sin m\omega t dt = \frac{1}{2} \int_t^{t+T_0} 1 dt = \frac{1}{2} (t) \Big|_t^{t+T_0} = T_0/2$$

$$\int_t^{t+T_0} \sin n\omega t \sin m\omega t dt = \begin{cases} 0; & n \neq m \\ T_0/2; & n = m \end{cases}$$

iii, To check orthogonality of $\sin n\omega t$ & $\cos m\omega t$

$$\int_t^{t+T_0} \sin n\omega t \cos m\omega t dt$$

$$\int \sin x \cos y dt = \frac{1}{2} [\sin(x-y) + \sin(x+y)]$$

$$\int_t^{t+T_0} \sin n\omega t \cos m\omega t = \frac{1}{2} \int_t^{t+T_0} [\sin(n-m)\omega t + \sin(n+m)\omega t] dt$$

$$= \frac{1}{2} \int_t^{t+T_0} \sin(n-m)\omega t dt + \frac{1}{2} \int_t^{t+T_0} \sin(n+m)\omega t dt$$

Integration of $(n-m)$ or $(n+m)$ full cycles of sine wave over a period will be zero.

Hence, above both integrals are zero.

$$\int_t^{t+T_0} \sin n\omega t \cos m\omega t dt = 0 \text{ for all values } n \text{ \& } m$$

\therefore Thus, sin & cosine waves of a given set are orthogonal over one period.

prove that set of exponential, 1, $e^{\pm j\omega t}$, $e^{\pm 2j\omega t}$, $e^{\pm 3j\omega t}$, ... is orthogonal over an interval T_0 .

Q.1 Hence we have to check orthogonality of complex function and is given as

$$\int_t^{t+T_0} x_m(t) x_n^*(t) dt = \begin{cases} 0; & m \neq n \\ T_0; & m = n \end{cases}$$

$$\text{for } x_m(t) = e^{jm\omega t}, \quad x_n^*(t) = e^{-jn\omega t}$$

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$$\int_t^{t+T_0} e^{jm\omega_0 t} [e^{jn\omega_0 t}]^* dt = \int_t^{t+T_0} e^{jm\omega_0 t} e^{-jn\omega_0 t} dt \rightarrow 0$$

$$= \int_t^{t+T_0} e^{j(m-n)\omega_0 t} dt \quad \left[\int e^x = \frac{1}{x} e^x \right]$$

$$\int_t^{t+T_0} e^{j(m-n)\omega_0 t} dt = \frac{1}{j(m-n)\omega_0} \left[e^{j(m-n)\omega_0 t} \right]_t^{t+T_0}$$

$$= \frac{1}{j(m-n)\omega_0} \left[e^{j(m-n)\omega_0(t+T_0)} - e^{j(m-n)\omega_0 t} \right]$$

$$= \frac{1}{j(m-n)\omega_0} \left[e^{j(m-n)\omega_0 t} e^{j(m-n)\omega_0 T_0} - e^{j(m-n)\omega_0 t} \right]$$

$$= \frac{1}{j(m-n)\omega_0} e^{j(m-n)\omega_0 t} \left[e^{j(m-n)\omega_0 T_0} - 1 \right]$$

② \Rightarrow

$$\int_t^{t+T_0} e^{jm\omega_0 t} [e^{jn\omega_0 t}]^* dt = \frac{1}{j(m-n)\omega_0} e^{j(m-n)\omega_0 t} \left[e^{j(m-n)2\pi} - 1 \right]$$

$$\left[\because e^{j(m-n)2\pi} = 1 \right]$$

WKT, $e^{j\theta} = \cos\theta + j\sin\theta$

$$= \cos 2\pi + j\sin 2\pi$$

$$= \frac{1}{j(m-n)\omega_0} e^{j(m-n)\omega_0 t} [1 - 1]$$

$$= \cos(m-n) 2\pi + j\sin(m-n) 2\pi = 0.$$

Thus, complex exponentials are orthogonal over any time period T_0 . When $n \neq m$.

$$\int_t^{t+T_0} e^{j(m-n)\omega_0 t} dt \int_t^{t+T_0} j m \omega_0 [e^{j n \omega_0 t}]^* dt$$

$$= \int_t^{t+T_0} e^{j(m-m)\omega_0 t} dt = \int_t^{t+T_0} 1 dt$$

$$= [t]_t^{t+T_0}$$

$$= \cancel{t+T_0} - \cancel{t} = T_0$$

$$\int_t^{t+T_0} e^{j(m-n)\omega_0 t} dt \int_t^{t+T_0} j m \omega_0 [e^{j n \omega_0 t}]^* dt = \begin{cases} 0; n \neq m \\ T_0; n = m \end{cases}$$

Q) Show that $e^{j2\pi k n/N}$ is an orthogonal sequence, periodic is N .

Sol) periodicity:

consider $x(n) = e^{j2\pi k n/N}$

It will be periodic if $x(n+N) = x(n)$ [$\because e^{j2\pi k} = 1$]

$$x(n+N) = e^{j2\pi k (n+N)/N} = e^{j2\pi k n/N} e^{j2\pi k}$$

Hence, $e^{j2\pi k} = \cos 2\pi k + j \sin 2\pi k = 1$; for all values of k .

$$x(n+N) = e^{j2\pi k n/N} = x(n)$$

$\therefore x(n)$ is periodic with period N .

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To check for orthogonality:

consider two sequences $x_k(n) = e^{j2\pi kn/N}$ & $x_l(n) = e^{j2\pi ln/N}$

orthogonality of discrete type sequences can be checked over one period.

$$\begin{aligned} \text{i.e. } \sum_{n=0}^{N-1} x_k(n) x_l^*(n) &= \sum_{n=0}^{N-1} e^{j2\pi kn/N} [e^{j2\pi ln/N}]^* \\ &= \sum_{n=0}^{N-1} e^{j2\pi kn/N} e^{-j2\pi ln/N} \\ &= \sum_{n=0}^{N-1} e^{j2\pi \frac{(k-l)n}{N}} \end{aligned}$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_2+1} - a^{N_1+1}}{1-a} \quad (N_2 > N_1)$$

$$\text{Here } a = e^{j2\pi \frac{(k-l)}{N}}$$

$$\sum_{n=0}^{N-1} x_k(n) x_l^*(n) = \frac{[e^{j2\pi \frac{(k-l)}{N}}]^0 - [e^{j2\pi \frac{(k-l)}{N}}]^{N-k+l}}{1 - e^{j2\pi \frac{(k-l)}{N}}}$$

$$= \frac{1 - e^{j2\pi \frac{(k-l)}{N}}}{1 - e^{j2\pi \frac{(k-l)}{N}}}$$

$$= \frac{1 - e^{j2\pi (k-l)}}{1 - e^{j2\pi \frac{(k-l)}{N}}}$$

Here k & l are integers, so $k-l$ will also be an integer.

$$e^{j2\pi(k-l)} = 1$$

$$\sum_{n=0}^{N-1} x_k(n) x_l^*(n) = \frac{1-1}{1 - e^{j2\pi \left(\frac{k-l}{N}\right)}} = 0.$$

check the orthogonality of $x_1(n) = e^{jk(\pi/8)n}$ &
 $x_2(n) = e^{jm(\pi/8)n}$.

Sol:

consider $x_1(n) = e^{jk(\pi/8)n}$, where $x_1(n) = e^{j2k(\pi/16)n}$

$x_2(n) = e^{jm(\pi/8)n}$, where $x_2(n) = e^{j2m(\pi/16)n}$.

$$\sum_{n=0}^{N-1} x_1(n) x_2^*(n) = \sum_{n=0}^{N-1} e^{j2k(\pi/16)n} [e^{j2m(\pi/16)n}]^*$$

$$= \sum_{n=0}^{N-1} e^{j2k(\pi/16)n} e^{-j2m(\pi/16)n}$$

$$= \sum_{n=0}^{N-1} e^{j2\pi \left(\frac{k-m}{16}\right)n}$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1} - a^{N_2+1}}{1-a} \quad (N_2 > N_1); \quad a = \frac{e^{j2\pi(k-m)}}{16}$$

$$\sum_{n=0}^{N-1} x_1(n) x_2^*(n) = \frac{\left[e^{j2\pi \left(\frac{k-m}{16}\right)n} \right]_0^{N-1}}{1 - e^{j2\pi \left(\frac{k-m}{16}\right)}} = \frac{\left[e^{j2\pi \left(\frac{k-m}{16}\right)n} \right]_0^{N-1}}{1 - e^{j2\pi \left(\frac{k-m}{16}\right)}}$$

$$\sum_{n=0}^{N-1} x_1(n) x_2^*(n) = \frac{1 - e^{j2\pi \left(\frac{k-m}{16}\right)N}}{1 - e^{j2\pi \left(\frac{k-m}{16}\right)}} \quad (\because N=16)$$

$$= \frac{1 - e^{j2\pi \frac{k-m}{16}}}{1 - e^{j2\pi \frac{k-m}{16}}} \quad (\because \text{where } e^{j2\pi(k-m)} = 1)$$

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$$\sum_{n=0}^{N-1} a_1(n) a_2^*(n) = \frac{1-1}{1 - e^{j2\pi \frac{(k-m)}{L}}} = 0$$

∴ Two signals are orthogonal